

$$(3) \sqrt{9} < \sqrt{12} < \sqrt{16} \text{ であるから } 3 < \sqrt{12} < 4$$

$$\text{よって } a=3, \quad b=\sqrt{12}-3$$

$$\begin{aligned} \text{ゆえに } a^2 - b^2 &= 3^2 - (\sqrt{12}-3)^2 \\ &= 9 - (12 - 6\sqrt{12} + 9) \\ &= -12 + 6\sqrt{12} \\ &= -12 + 12\sqrt{3} \end{aligned}$$

$$(4) \sqrt{4} < \sqrt{6} < \sqrt{9} \text{ であるから } 2 < \sqrt{6} < 3$$

$$\text{よって } a=2, \quad b=\sqrt{6}-2$$

$$\begin{aligned} \text{ゆえに } a^3b + ab^3 &= ab(a^2 + b^2) \\ &= 2(\sqrt{6}-2)\{2^2 + (\sqrt{6}-2)^2\} \\ &= (2\sqrt{6}-4)\{4 + (6 - 4\sqrt{6} + 4)\} \\ &= (2\sqrt{6}-4)(14 - 4\sqrt{6}) \\ &= 28\sqrt{6} - 48 - 56 + 16\sqrt{6} \\ &= -104 + 44\sqrt{6} \end{aligned}$$

$$\begin{aligned} 108 \quad (1) \quad & \sqrt{6} + \sqrt{27} - \frac{\sqrt{18}-6}{\sqrt{3}} \\ &= \sqrt{6} + 3\sqrt{3} - \frac{(\sqrt{18}-6) \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} \\ &= \sqrt{6} + 3\sqrt{3} - \frac{\sqrt{54}-6\sqrt{3}}{3} \\ &= \sqrt{6} + 3\sqrt{3} - \frac{3\sqrt{6}-6\sqrt{3}}{3} \\ &= \sqrt{6} + 3\sqrt{3} - \sqrt{6} + 2\sqrt{3} \\ &= 5\sqrt{3} \end{aligned}$$

$$\begin{aligned} (2) \quad & \sqrt{2} - \sqrt{3}(\sqrt{6}-2) - \frac{6}{\sqrt{3}} \\ &= \sqrt{2} - \sqrt{18} + 2\sqrt{3} - \frac{6 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} \\ &= \sqrt{2} - 3\sqrt{2} + 2\sqrt{3} - \frac{6\sqrt{3}}{3} \\ &= \sqrt{2} - 3\sqrt{2} + 2\sqrt{3} - 2\sqrt{3} \\ &= -2\sqrt{2} \end{aligned}$$

$$\begin{aligned} (3) \quad & \frac{6\sqrt{3}}{\sqrt{6}} + \frac{\sqrt{72}}{3}(\sqrt{2}-1) \\ &= \frac{6\sqrt{3} \times \sqrt{6}}{\sqrt{6} \times \sqrt{6}} + \frac{6\sqrt{2}}{3}(\sqrt{2}-1) \\ &= \frac{6\sqrt{18}}{6} + 2\sqrt{2}(\sqrt{2}-1) \\ &= \sqrt{18} + 2 \times 2 - 2\sqrt{2} \\ &= 3\sqrt{2} + 4 - 2\sqrt{2} \\ &= 4 + \sqrt{2} \end{aligned}$$

$$\begin{aligned} (4) \quad & \frac{\sqrt{45}-\sqrt{27}}{\sqrt{3}(\sqrt{5}-\sqrt{3})} \\ &= \frac{3\sqrt{5}-3\sqrt{3}}{\sqrt{3}(\sqrt{5}-\sqrt{3})} \end{aligned}$$

$$\begin{aligned} &= \frac{3(\sqrt{5}-\sqrt{3})}{\sqrt{3}(\sqrt{5}-\sqrt{3})} \\ &= \frac{3}{\sqrt{3}} \\ &= \sqrt{3} \end{aligned}$$

$$\begin{aligned} (5) \quad & \frac{\sqrt{2}+\sqrt{6}}{\sqrt{8}} - \frac{\sqrt{5}-\sqrt{15}}{\sqrt{20}} \\ &= \frac{\sqrt{2}}{\sqrt{8}} + \frac{\sqrt{6}}{\sqrt{8}} - \frac{\sqrt{5}}{\sqrt{20}} + \frac{\sqrt{15}}{\sqrt{20}} \\ &= \sqrt{\frac{2}{8}} + \sqrt{\frac{6}{8}} - \sqrt{\frac{5}{20}} + \sqrt{\frac{15}{20}} \\ &= \sqrt{\frac{1}{4}} + \sqrt{\frac{3}{4}} - \sqrt{\frac{1}{4}} + \sqrt{\frac{3}{4}} \\ &= 2\sqrt{\frac{3}{4}} \\ &= \sqrt{3} \end{aligned}$$

$$\begin{aligned} (6) \quad & \frac{\sqrt{5}(\sqrt{10}+3)}{5} - \frac{3+\sqrt{20}}{\sqrt{5}} + \frac{\sqrt{8}}{\sqrt{2}} \\ &= \frac{\sqrt{5}(\sqrt{10}+3)}{5} - \frac{(3+\sqrt{20}) \times \sqrt{5}}{\sqrt{5} \times \sqrt{5}} + \sqrt{\frac{8}{2}} \\ &= \frac{\sqrt{50}+3\sqrt{5}}{5} - \frac{3\sqrt{5}+\sqrt{100}}{5} + \sqrt{4} \\ &= \frac{\sqrt{50}-\sqrt{100}}{5} + \sqrt{4} \\ &= \frac{5\sqrt{2}-10}{5} + 2 \\ &= \sqrt{2} - 2 + 2 \\ &= \sqrt{2} \end{aligned}$$

$$\begin{aligned} (7) \quad & (3+2\sqrt{2})(3-2\sqrt{2}) + \frac{1}{\sqrt{12}}(1-\sqrt{3})^2 \\ &= 3^2 - (2\sqrt{2})^2 + \frac{1}{2\sqrt{3}}(1-2\sqrt{3}+3) \\ &= 9-8 + \frac{1}{2\sqrt{3}}(4-2\sqrt{3}) \\ &= 1 + \frac{4}{2\sqrt{3}} - \frac{2\sqrt{3}}{2\sqrt{3}} \\ &= 1 + \frac{2}{\sqrt{3}} - 1 \\ &= \frac{2\sqrt{3}}{3} \end{aligned}$$

$$\begin{aligned} (8) \quad & (\sqrt{2}+2\sqrt{3})(\sqrt{12}-\sqrt{2}) - \left(\frac{2\sqrt{3}}{\sqrt{2}}-2\right)^2 \\ &= (\sqrt{2}+2\sqrt{3})(2\sqrt{3}-\sqrt{2}) - \left(\frac{2\sqrt{3} \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} - 2\right)^2 \\ &= (2\sqrt{3}+\sqrt{2})(2\sqrt{3}-\sqrt{2}) - (\sqrt{6}-2)^2 \\ &= (2\sqrt{3})^2 - (\sqrt{2})^2 - (6-4\sqrt{6}+4) \\ &= 12-2-10+4\sqrt{6} \\ &= 4\sqrt{6} \end{aligned}$$

$$\begin{aligned} 109 \quad (1) \quad & (5+\sqrt{2})^2 - (\sqrt{2}-5)^2 \\ &= \{(5+\sqrt{2})+(\sqrt{2}-5)\}\{(5+\sqrt{2})-(\sqrt{2}-5)\} \\ &= 2\sqrt{2} \times 10 \\ &= 20\sqrt{2} \\ (2) \quad & (\sqrt{3}+2+\sqrt{6})^2 - (\sqrt{3}-2+\sqrt{6})^2 \\ &= \{(\sqrt{3}+2+\sqrt{6})+(\sqrt{3}-2+\sqrt{6})\} \\ & \quad \times \{(\sqrt{3}+2+\sqrt{6})-(\sqrt{3}-2+\sqrt{6})\} \\ &= \{2(\sqrt{3}+\sqrt{6})\} \times 4 \\ &= 8\sqrt{3}+8\sqrt{6} \\ (3) \quad & (\sqrt{2}-\sqrt{6}-\sqrt{3})(\sqrt{2}-\sqrt{6}+\sqrt{3}) \\ &= \{(\sqrt{2}-\sqrt{6})-\sqrt{3}\}\{(\sqrt{2}-\sqrt{6})+\sqrt{3}\} \\ &= (\sqrt{2}-\sqrt{6})^2 - (\sqrt{3})^2 \\ &= 2-2\sqrt{12}+6-3 \\ &= 5-4\sqrt{3} \\ (4) \quad & (\sqrt{3}+\sqrt{2}+1)(\sqrt{3}-\sqrt{2}+1) \\ &= \{(\sqrt{3}+1)+\sqrt{2}\}\{(\sqrt{3}+1)-\sqrt{2}\} \\ &= (\sqrt{3}+1)^2 - (\sqrt{2})^2 \\ &= 3+2\sqrt{3}+1-2 \\ &= 2+2\sqrt{3} \\ (5) \quad & (\sqrt{3}-\sqrt{2})(5+\sqrt{6})(\sqrt{2}+\sqrt{3})(-5+\sqrt{6}) \\ &= (\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})(\sqrt{6}+5)(\sqrt{6}-5) \\ &= \{(\sqrt{3})^2 - (\sqrt{2})^2\}\{(\sqrt{6})^2 - 5^2\} \\ &= (3-2)(6-25) \\ &= -19 \end{aligned}$$

$$\begin{aligned} 110 \quad (1) \quad & x^2 - y^2 \\ &= (x+y)(x-y) \\ &= \left(\frac{\sqrt{5}-1}{\sqrt{3}} + \frac{\sqrt{5}+1}{\sqrt{3}}\right) \left(\frac{\sqrt{5}-1}{\sqrt{3}} - \frac{\sqrt{5}+1}{\sqrt{3}}\right) \\ &= \frac{2\sqrt{5}}{\sqrt{3}} \times \left(-\frac{2}{\sqrt{3}}\right) \\ &= -\frac{4\sqrt{5}}{3} \end{aligned}$$

$$\begin{aligned} (2) \quad & x^2 - 6xy + y^2 \\ &= (x+y)^2 - 8xy \\ &= \left(\frac{1+\sqrt{5}}{2} + \frac{1-\sqrt{5}}{2}\right)^2 - 8 \times \frac{1+\sqrt{5}}{2} \times \frac{1-\sqrt{5}}{2} \\ &= 1^2 - 2 \times (1+\sqrt{5}) \times (1-\sqrt{5}) \\ &= 1 - 2 \times (1-5) \\ &= 9 \end{aligned}$$

$$\begin{aligned} (3) \quad & x^2 + xy + y^2 \\ &= (x+y)^2 - xy \\ &= \left(\frac{\sqrt{2}-1}{3} + \frac{\sqrt{2}+1}{3}\right)^2 - \frac{\sqrt{2}-1}{3} \times \frac{\sqrt{2}+1}{3} \\ &= \left(\frac{2\sqrt{2}}{3}\right)^2 - \frac{(\sqrt{2}-1)(\sqrt{2}+1)}{9} \\ &= \frac{8}{9} - \frac{2-1}{9} = \frac{7}{9} \end{aligned}$$

$$\begin{aligned} 111 \quad (1) \quad & x=4+\sqrt{3} \text{ より } x-4=\sqrt{3} \\ \text{両辺を2乗すると } & (x-4)^2 = (\sqrt{3})^2 \\ \text{すなわち } & x^2 - 8x + 16 = 3 \\ \text{したがって } & x^2 - 8x = -13 \\ \text{別解 } & x^2 - 8x = x(x-8) \\ &= (4+\sqrt{3}) \times \{(4+\sqrt{3})-8\} \\ &= (4+\sqrt{3}) \times (-4+\sqrt{3}) \\ &= (\sqrt{3}+4) \times (\sqrt{3}-4) \\ &= (\sqrt{3})^2 - 4^2 \\ &= 3-16 \\ &= -13 \end{aligned}$$

$$(2) \quad x = \frac{1-3\sqrt{5}}{2} \text{ より } 2x-1 = -3\sqrt{5}$$

$$\begin{aligned} \text{両辺を2乗すると } & (2x-1)^2 = (-3\sqrt{5})^2 \\ \text{すなわち } & 4x^2 - 4x + 1 = 45 \\ \text{よって } & 4x^2 - 4x = 44 \\ \text{したがって } & x^2 - x = 11 \end{aligned}$$

$$\begin{aligned} \text{別解 } & x^2 - x = x(x-1) \\ &= \frac{1-3\sqrt{5}}{2} \times \left(\frac{1-3\sqrt{5}}{2} - 1\right) \\ &= \frac{1-3\sqrt{5}}{2} \times \frac{-1-3\sqrt{5}}{2} \\ &= -\left(\frac{1-3\sqrt{5}}{2} \times \frac{1+3\sqrt{5}}{2}\right) \\ &= -\frac{1^2 - (3\sqrt{5})^2}{4} \\ &= -\frac{1-45}{4} \\ &= 11 \end{aligned}$$

$$\begin{aligned} 112 \quad (1) \quad & \sqrt{4} < \sqrt{7} < \sqrt{9} \text{ であるから } 2 < \sqrt{7} < 3 \\ \text{よって } & a=2, \quad b=\sqrt{7}-2 \\ \text{このとき } & a^2 + 2ab + 2b^2 = (a+b)^2 + b^2 \end{aligned}$$

$$\begin{aligned} &= (\sqrt{7})^2 + (\sqrt{7}-2)^2 \\ &= 7 + 7 - 4\sqrt{7} + 4 \\ &= 18 - 4\sqrt{7} \end{aligned}$$

$$\begin{aligned} (2) \quad & \sqrt{1} < \sqrt{2} < \sqrt{4} \text{ であるから } 1 < \sqrt{2} < 2 \\ \text{よって } & -2 < -\sqrt{2} < -1 \\ \text{ゆえに } & 1 < 3 - \sqrt{2} < 2 \\ \text{したがって } & a=1, \quad b=(3-\sqrt{2})-1=2-\sqrt{2} \\ \text{よって } & a + \frac{8}{b^2-6} = 1 + \frac{8}{(2-\sqrt{2})^2-6} \\ &= 1 + \frac{8}{4-4\sqrt{2}+2-6} \\ &= 1 + \frac{8}{-4\sqrt{2}} = 1 - \frac{2}{\sqrt{2}} \\ &= 1 - \sqrt{2} \end{aligned}$$